

NOTES

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Viscosity of Water Vapor in the Temperature Range from 6°C to 29°C

Isao YASUMOTO

Department of Chemistry, Yonago Technical College, Hikona-cho, Yonago

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The viscosities of gases^{1,2)} have already been measured over a wide range of temperatures and pressures. The viscosity of water vapor^{3,4)} at room temperature has, however, scarcely been measured because of the technical difficulties involved. This paper will present the results of our viscosity measurement of water vapor in the temperature range from 6 to 29°C by means of a new tandem capillary-flow viscometer.

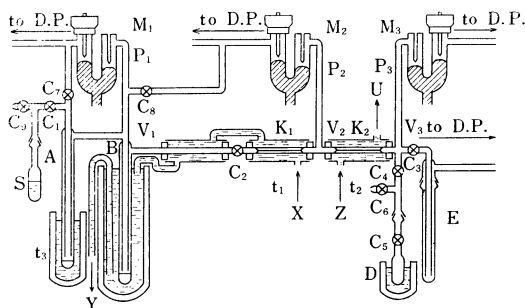


Fig. 1. Schematic diagram of the apparatus used for viscosity measurement of water vapor. The main part of the apparatus is constructed of borosilicate glass.

The apparatus used (Fig. 1) consists of a vapor-supply system, S; two traps, A and B; two capillaries, K_1 and K_2 , in tandem; three vacuum gauges, M 's; and a water-vapor collector, D. The main capillary, K_1 , is 1.207 mm in diameter and 493.5 mm in length, while the auxiliary capillary, K_2 , is about 1.2 mm in diameter and about 1.2 meter in length. The vacuum gauges are essential-

ly the same as the mercury U-tube manometer devised by Meyer and Wade.⁵⁾ The details of the manometer as modified by the present author have already been published in his other report.⁶⁾

The whole apparatus was evacuated until a pressure below 1×10^{-5} mmHg was attained. In carrying out a run, water was first freed from the dissolved gases by vacuum sublimation from the water reservoir, S, to the trap B, immersed in a dry ice-acetone mixture. The amount of water trapped there was about 10 cc. Then, by circulating water of a constant temperature from X to Y and from Z to U, the temperature of B and K_1 was held constant at t_1 , and that of K_2 , at t_2 . The trap A was immersed in water of a constant temperature, t_3 , approximately 0.5°C lower than t_1 , in order to prevent the water vapor in the space V_1 from saturating and eventually condensing in the capillary tube K_1 .

As the water vapor in the space V_1 flowed through the capillaries, K_1 and K_2 , the pressure difference, $P_1 - P_2$, was slowly developed across the capillary K_1 to become stationary after about 20 minutes. As a consequence of attaching K_2 in tandem, a laminar flow was attained throughout the capillary K_1 . When the cock C_5 was turned on and C_3 closed, the water vapor in the space V_3 began to be condensed in the collector, D, cooled by liquid nitrogen, the pressures of P_1 , P_2 , and P_3 remaining practically unaltered. After condensation had proceeded for a certain length of time, C_5 was closed. The weight increase in the collector, D, was then determined.

The weight increase due to mercury and grease vapors condensed in D during a run was ascertained to be negligible. The influence of the total vacuum leak of this apparatus was also negligible, since it was less than 1×10^{-5} mmHg·l/hr. No correction for the thermal expansion of the capillary tube K_1 was made, since the temperature range covered was narrow.

The results of all the experiments performed

1) G. P. Flynn, R. V. Hanks, N. A. Lemaire and J. Ross, *J. Chem. Phys.*, **38**, 154 (1963).

2) A. K. Barua, M. Afzal, G. P. Flynn and J. Ross, *ibid.*, **41**, 374 (1964).

3) H. Braune and R. Linke, *Z. Phys. Chem.*, **A148**, 195 (1930).

4) J. Kestin and W. Leidenfrost, *Physica*, **25**, 1033 (1959).

5) D. E. Meyer and W. H. Wade, *Rev. Sci. Instr.*, **33**, 1283 (1962).

6) I. Yasumoto, *J. Vac. Soc. Japan*, **13**, 232 (1970).

TABLE 1. RESULTS OF ALL THE EXPERIMENTS AND VISCOSITIES CALCULATED BY THE VARIOUS EQUATIONS

t_1^{*1} °C	P_1^{*2} mmHg at 0°C	P_2^{*2} mmHg at 0°C	Flow rate g/sec ($\times 10^5$)	η_1^{*3} poise ($\times 10^7$)	η_2^{*4} poise ($\times 10^7$)	η_3^{*5} poise ($\times 10^7$)	η_4^{*6} poise ($\times 10^7$)	η_5^{*7} poise ($\times 10^7$)
6.73	6.51	5.31	1.1646	887	913	901.9	907.6	843.7
7.18	6.97	5.61	1.4104	881	905	903.6	909.4	845.4
10.20	8.58	6.92	2.0476	907	927	915.0	921.1	857.1
13.10	10.48	8.45	2.9629	923	940	925.9	932.4	868.2
15.45	12.84	10.42	4.2595	932	946	934.8	941.5	877.3
17.11	13.47	10.93	4.6504	932	946	941.1	948.0	883.7
17.74	14.56	11.81	5.4267	935	948	943.4	950.4	886.1
19.90	15.20	12.31	5.8713	940	952	951.6	958.8	894.5
20.53	16.32	13.19	6.7866	942	954	954.0	961.3	896.9
22.27	17.61	14.22	7.8114	951	962	960.6	968.1	903.6
23.38	19.29	15.64	9.2427	947	958	964.8	972.4	907.9
24.03	19.52	15.84	9.3358	953	963	967.3	974.9	910.4
27.13	22.60	18.37	12.173	964	973	979.1	987.0	922.5
27.17	21.29	17.16	11.156	966	976	979.2	987.1	922.6
29.00	21.99	17.68	11.835	972	982	986.2	994.3	929.7
100.0					1270 ^{*8}	1261	1269	1208
151.2					1450 ^{*8}	1465	1466	1410
207.1					1680 ^{*8}	1693	1678	1632
261.3					1900 ^{*8}	1918	1880	1847
20.0					937.3 ^{*9}	952.2	959.4	895.0
92.4					1178 ^{*10}	1231	1240	1178
107.3					1242 ^{*10}	1290	1298	1237
210.1					1638 ^{*10}	1705	1689	1644
312.9					2049 ^{*10}	2136	2069	2050
366.2					2261 ^{*10}	2364	2260	2258
406.4					2422 ^{*10}	2539	2401	2414
20.00					967.42 ^{*11}			
24.91					969.40 ^{*11}			

*1 t_1 is an average of temperatures measured at two minute intervals through out a run. The fluctuation of t_1 was $\pm 0.08^\circ\text{C}$.

*2 P_1 and P_2 are averages of pressures measured at two minute intervals through out a run.

*3 Calculated by the Hagen-Poiseuille equation from the present author's data given in column 1, 2, 3 and 4.

*4 Corrected for slip.

*5 Calculated by the new viscosity formula proposed by the present author:

$$\eta_3 = KT^{7/6} = 126 \times 10^{-9} \times T^{7/6}.$$

*6 Calculated by the Sutherland's formula:

$$\eta_4 = KT^{3/2}/(T+C),$$

where C is 650 as is given by Smith. Using the Smith's data, K was calculated by the present author to be 180.25×10^{-7} .

*7 Calculated by the Sutherland's formula given by Braune and Linke:

$$\eta_5 = 223.6 \times 10^{-7} \times T^{3/2}/(T+961).$$

*8 Smith's experimental values.

*9 Braune and Linke's (saturated water vapor).

*10 Braune and Linke's.

*11 Kestin and Leidenfrost's.

are tabulated in Table 1. The viscosity, η_1 , of the water vapor, as calculated by the Hagen-Poiseuille equation, is shown in column 5. The viscosity, η_2 , corrected for slip is shown in column 6, together with the values obtained by other authors. The correction was made by multiplying η_1 by the factor of $(1+4\zeta/r)$, where r is the radius of the capillary tube K_1 and where ζ is 0.85λ . λ is an average of the mean free path

inside K_1 expressed by $\lambda = kT/(2^{1/2}\pi\sigma^2P)$, where σ is the molecular diameter, as will be discussed below, and where P is equal to $(P_1 + P_2)/2$, as was shown by Smith.⁷⁾ The precision of the viscosity, η_2 , obtained was estimated to be less than 0.5%.

A plot of the author's viscosities, η_2 's, against

7) C. J. Smith, *Proc. Roy. Soc.*, **A106**, 83 (1924).

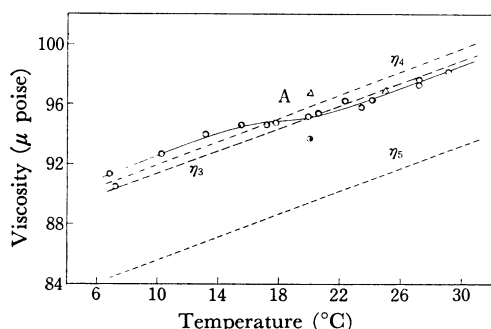


Fig. 2. ○; author's experimental values, ●; Braune and Linke's (saturated water vapor), △; Kestin and Leidenfrost's. Broken lines represent η_3 , η_4 and η_5 , respectively.

the temperature shows the inflection shown in Fig. 2. A curve similar in form to the part A of this curve was also observed by Rankine⁸⁾ at room temperature in his experiment using bromine vapor. One of the two values obtained by Kestin and Leidenfrost by the oscillating-disk method is in good agreement with the present author's.

In Table 1, η_3 is calculated by the one-constant formula derived by the author, which will be discussed later. Taken in connection with Fig. 2, this formula can represent remarkably well (within 1%) not only one half of the author's experimental results (one half above 17°C because of the inflection), but also Smith's, extending from 100 to 261°C. Sutherland's two-constant formula, with the C value given by Smith and with the K value given by the present author, can also represent excellently (within approximately 1%) the present author's and Smith's results as a whole. The former formula gives a curve slightly concave upward, while the latter gives a curve slightly concave downward, so the latter curve in the highest temperature region gradually deviates from the former to approach the plot of Braune and Linke's experimental values, (Fig. 3). This plot is a nearly straight line extending from 90 to 406°C, and it is nearly perfectly represented by Sutherland's formula, with the C and K values given by Braune and Linke. It is parallel to, and situated lower than, the plot of Smith's data by the amount of $ca. 50 \times 10^{-7}$ poise.

The derivation of the author's formula used for the calculation of η_3 is as follows: The potential function, ϕ , for water-molecule interaction is

8) A. O. Rankine, *ibid.*, **A88**, 575 (1913).

9) E. A. Moelwyn-Hughes, "Physical Chemistry," 2nd Revised Edition, Pergamon Press Limited, London (1961), p. 338.

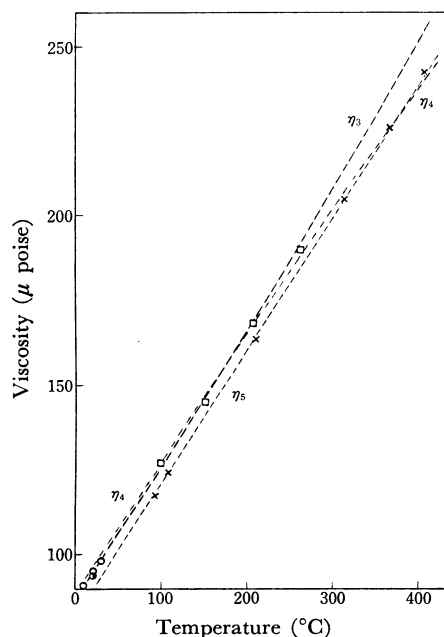


Fig. 3. ○: author's experimental values, ●: Braune and Linke's (saturated water vapor), ×: Braune and Linke's, □; Smith's. Broken lines represent η_3 , η_4 and η_5 , respectively.

represented by⁹⁾:

$$\phi = Aa^{-12} - Ba^{-6} - Ca^{-3},$$

where a is the distance between molecules and where A , B , and C are constants. Neglecting the first two terms, the approximate distance, a , in collision is deduced¹⁰⁾ to be:

$$a = (3C/2kT)^{1/3},$$

which we may refer to as the diameter of capture, and which we may identify with σ , the molecular diameter. Denoting the mass of a molecule by m , the expression for the viscosity becomes:

$$\begin{aligned} \eta &= 1/\pi \sigma^2 (mkT/\pi)^{1/2} \\ &= 1/\pi (mk/\pi)^{1/2} (2k/3C)^{2/3} T^{1/2 + 2/3} \\ &= KT^{7/6}. \end{aligned}$$

Plotting η_2 against $T^{7/6}$, K was found to be 126×10^{-9} , from which C was calculated to be 2.55×10^{-36} erg·cm³.

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10) *Ibid.*, p. 611.